Research Article

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Shear buckling of steel foam sandwich panel resting on Pasternak foundation

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Abstract: The objective of this paper is to assess the in-plane shear buckling of a steel foam sandwich panel that relies on elastic Pasternak foundation. The panel is a combination of solid steel face sheets and foamed steel cores. Foamed steel, that is steel with internal voids, provides enhanced bending rigidity and energy dissipation, and also, the potential to reduce local buckling. The Classic plate theory is employed where their governing equations are solved by the Rayleigh–Ritz method. Uniformly distributed in-plane shear loads are applied to the two opposite edges of the panel and all the four edges of the panel are simply supported. Finally, the effects of the panel parameters, such as the existence of a Pasternak foundation, aspect ratios, and central fraction of the steel foam core, are presented. The results showed that the optimum central fraction of the steel foam core would be 65%, so that the maximum critical shear buckling load has taken place.

Keywords: Shear buckling, steel foam, sandwich panel, Pasternak foundation

1 Introduction

Foam and cellular materials have been produced from base materials that include polymers, ceramics, and metals. The foam materials have been used to solve engineering problems in the aerospace, automotive, structures and so on. Steel is one of the most widely used engineering materials, but steel foam has not yet been used commercially [1].

Steel foam (Figure 1) is an isotropic and porous steel material with a cellular structure and extraordinary mechanical as well as physical properties at a very low density. A variety of industrial methods are used to produce the voids from powder metallurgy and sintering of hollow spheres to gasification. In general, steel foams have high bending rigidity and energy absorption. In addition, steel foams in comparison with solid steel have helped to improve fire resistance, noise attenuation, thermal conductivity, and provide improved electromagnetic and radiation shielding [2].

The conventional sandwich structures are ordinarily manufactured from three isotropic layers, with two face sheets tenaciously bonded to the core. The base of the sandwich structure theory has been covered in the literature [3, 4]. Several investigations have been carried out in the field of local buckling of sandwich structures [5, 6]. Szyniszewski *et al.* [2] prepared and verified a new design method for the in-plane compressive strength of the steel foam sandwich panel. Magnucka [7] studied the dynamic stability of a metal foam circular plate using the Hamilton principle. The government equation was numerically solved so that critical loads could be determined. The effect of porosity of the plate on the critical loads was shown.

For some structural reasons, it is possible that the panel rests on elastic foundation. Various types of elastic foundation models are used to consider the plate–foundation interactions. Winkler [8] proposed a one-parameter model for plate-foundation interactions [9]. The model is assumed to be a combination of unconnected independent linear springs that are close to each other. Pasternak [10] improved the Winkler model by adding a shear layer. In the Pasternak model, as an improved two-parameter model, the shear interaction between the independent springs are modeled by connecting the ends of the springs to a beam or plate that only tolerates transverse shear deformation. The mechanical behavior of structure–foundation interactions is widely described by the model [11, 12].

The objective of this study is to assess the in-plane shear buckling of steel foam sandwich panels that is resting on elastic Pasternak foundation. The Classic panel theory is employed where their governing equations are solved by the Rayleigh–Ritz method. Uniformly distributed in-plane shear loads are applied to two opposite

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edges of the panel and all the four edges of the panel are simply supported. Finally, the effects of the panel parameters such as the existence of a Pasternak foundation, aspect ratios, and central fraction of the steel foam core are presented.

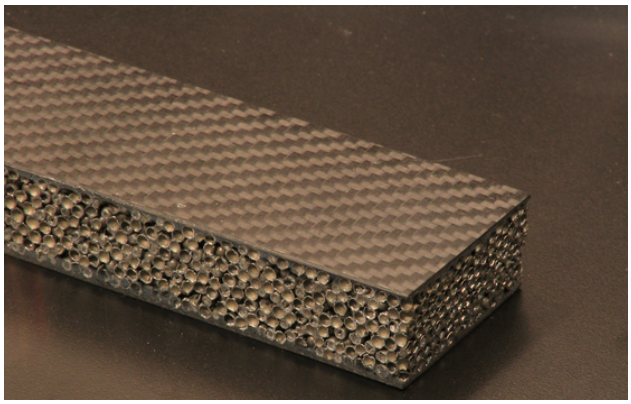


Figure 1: Steel foam sandwich panels

2 Governing equations

Consider an orthotropic panel and in-plane dimensions of a and b resting on Pasternak elastic foundation as shown in Figure 2.

The governing energy equation of the orthotropic panel resting on a Pasternak foundation under uniform in-plane shear edge load, N_{xy}^0, can be written as shown below. However, in the equation, the axial in-plane displacements have been neglected.

$$\Pi = \frac{1}{2}\int_0^b\int_0^a\left[D_{11}\left(\frac{d^2w_0}{dx^2}\right)^2 + 2D_{12}\frac{d^2w_0}{dx^2}\frac{d^2w_0}{dy^2} + D_{22}\left(\frac{d^2w_0}{dy^2}\right)^2 + 4D_{66}\left(\frac{d^2w_0}{dxdy}\right)^2 + k_w{w_0}^2 + k_s\left(\left(\frac{dw_0}{dx}\right)^2 + \left(\frac{dw_0}{dy}\right)^2\right) - N_{xy}^0\frac{dw_0}{dx}\frac{dw_0}{dy}\right]dxdy \tag{1}$$

where w_0 is the transverse displacement of the panel, k_w is the vertical spring modulus of the foundation and k_s is the shear modulus of the foundation. D_{ij} is the flexural stiffness matrix, as given in the following equation.

$$E_{bx} = \frac{E_x}{1-\nu_{xy}.\nu_{yx}}, \quad E_{by} = \frac{E_y}{1-\nu_{xy}.\nu_{yx}}, \tag{2}$$
$$D_{11} = \frac{h^3}{12}E_{bx}, \quad D_{12} = \frac{h^3}{12}E_{by}\nu_{xy},$$
$$D_{22} = \frac{h^3}{12}E_{by}, \quad D_{66} = \frac{h^3}{12}G_{xy},$$

where E_x and E_y are elastic modulus of the panel, ν_{xy} and ν_{yx} are Poisson's ratio of panel, G_{xy} is shear modulus of panel.

However, it can be considered that steel foam is isotropic. Consider a panel with initial thickness t_{ini}, if the whole panel is foamed, the thickness t_f is [12]:

$$t_f = t_{ini}/\rho \tag{3}$$

where ρ is the relative density of the foamed steel; thus, $\rho = 1$ is a solid steel panel. Based on the work of [12], the steel foam moduli, E_f and G_f are related to the solid steel moduli, E_s and G_s, respectively, by:

$$E_f \approx E_s\rho^2 \; G_f \approx \frac{3}{8}G_s\rho^2 \tag{4}$$

If, instead of foaming the entire panel, it is possible to foam only a central fraction of the core, $\alpha(0 \le \alpha \le 1)$, then it would result in the production of steel foam sandwich panel and an increase in the panel bending rigidity. Now, assuming that the relative density ρ applies only to the foamed core, then the core thickness t_c that increases from the initial solid panel thickness t_{ini}, is:

$$t_c = \frac{\alpha t_{ini}}{\rho} \tag{5}$$

The remaining portion of the initial solid sheet is split equally between the two face sheets of thickness, t_s:

$$t_s = \frac{1-\alpha}{2}t_{ini} \tag{6}$$

The panel bending rigidity can be expressed as follows:

$$D_p = \frac{E_s t_s (t_c + t_s)^2}{2(1-\nu_s^2)} \tag{7}$$

Thus, Eq. (1) can be converted as shown below:

$$\Pi_i = \frac{D_p}{2}\int_0^b\int_0^a\left[\left(\frac{d^2w_0}{dx^2} + \frac{d^2w_0}{dy^2}\right)^2 - 2(1-\nu)\left\{\frac{d^2w_0}{dx^2}\frac{d^2w_0}{dy^2} - \frac{3}{8}\left(\frac{d^2w_0}{dxdy}\right)^2\right\}\right]dxdy \tag{8}$$
$$\Pi_e = \frac{1}{2}\int_0^b\int_0^a\left[-k_w{w_0}^2 - k_s\left(\left(\frac{dw_0}{dx}\right)^2 + \left(\frac{dw_0}{dy}\right)^2\right) - 2N_{xy}^0\frac{dw_0}{dx}\frac{dw_0}{dy}\right]dxdy$$
$$\Pi = \Pi_i + \Pi_e$$

substituting the proper out-of-plane displacement shape function into Eq. (8), the standard eigenvalue problem of

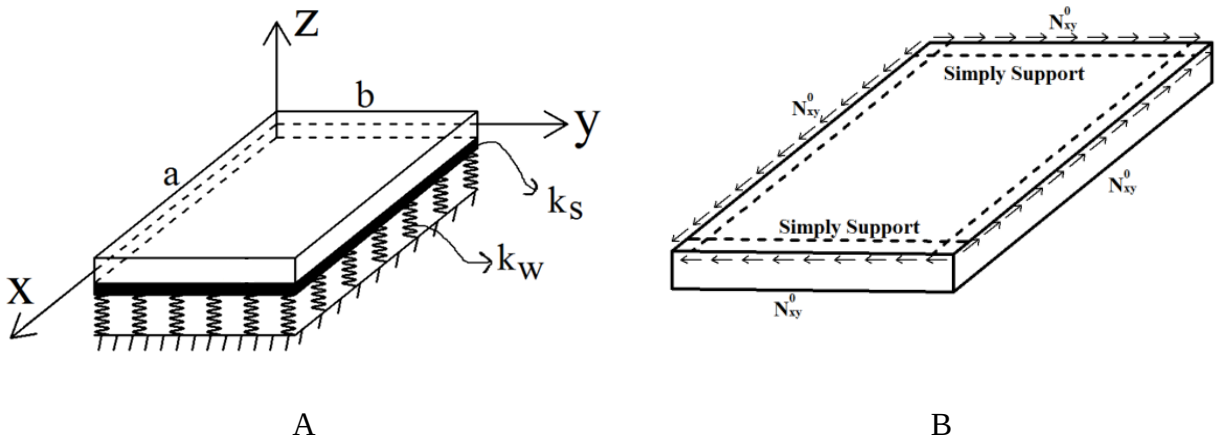


Figure 2: (A) Steel foam sandwich panels on foundation. (B) In-plane shear loading and boundary conditions

Table 1: Shear buckling in steel foamed panel with and without Pasternak foundation effect

α	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75	0.8
N_{xy}	1009	1334	1692	2068	2450	2821	3170	3480	3739	3931	4043	4060	3969	3754	3402
φ	64	63	62	61	61	60	60	60	60	60	60	60	60	60	60

buckling can be solved by the Rayleigh–Ritz method. The shape function must satisfy all boundary conditions. w_0 is the transverse displacement, which satisfies the boundary conditions, and W is the unknown constant that will remain indeterminate according to the Buckling Theory. The transverse displacement function is considered as following:

$$w_0 = W \sin\left(\frac{\pi(mx - \varphi y)}{a}\right) \sin\left(\frac{\pi m x}{a}\right) \sin\left(\frac{\pi y}{b}\right) \quad (9)$$

where m is the buckling half-waves and φ is the skew of the buckling mode. Using the equilibrium condition of the first variational principle of the total potential energy ($\delta\Pi = 0$); therefore, the buckling condition reduces to the well-known Ritz equations:

$$\frac{d\Pi}{dW} = 0 \quad (10)$$

3 Results and discussion

In this study, a steel foamed sandwich panel with dimension $a = 400$ mm and $b = 400$ mm was considered. Other characteristic panel is as follows:

$$t_{ini} = 5\ mm,\ \rho = 0.18,\ E_s = 2 * 10^5\ MPa\ \nu = 0.3$$

For the Pasternak effect, the Pasternak stiffness will be $k_w = 0.04$ N/mm^3 and $k_s = 50$ N/mm. Figure 3 shows the

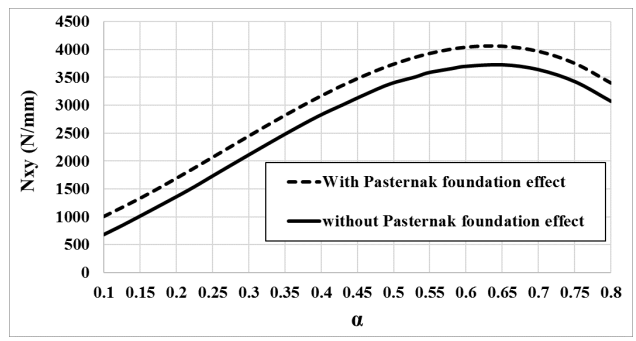


Figure 3: Critical shear buckling versus central fraction of the steel foam core with and without Pasternak foundation effect

critical shear buckling versus central fraction of the steel foam core (α) with and without the Pasternak foundation effect. It can be concluded that the optimum α is approximately 0.65, so that the maximum critical shear buckling load has taken place. In the absence of Pasternak foundation effect, for all α, the buckling half-wave is 1 and φ is 57°. It is necessary to mention that the aspect ratio is 1. Existing Pasternak foundation causes an increase in critical shear buckling. Surely, by increasing the foundation stiffness, the buckling load will increase. Table 1 shows the values of φcorresponding to critical shear buckling for various α, in the cases with and without Pasternak foundation effect.

Now for $\alpha = 0.65$, the results of shear buckling in the steel foamed sandwich panel with and without Pasternak

Table 2: Shear buckling in steel foamed panel with and without Pasternak foundation effect

	N_{xy}^0 (N/mm)	φ (degree)	Buckling half-waves
Steel foamed without Pasternak effect	3720	57	1
Steel foamed with Pasternak effect	4060	60	1

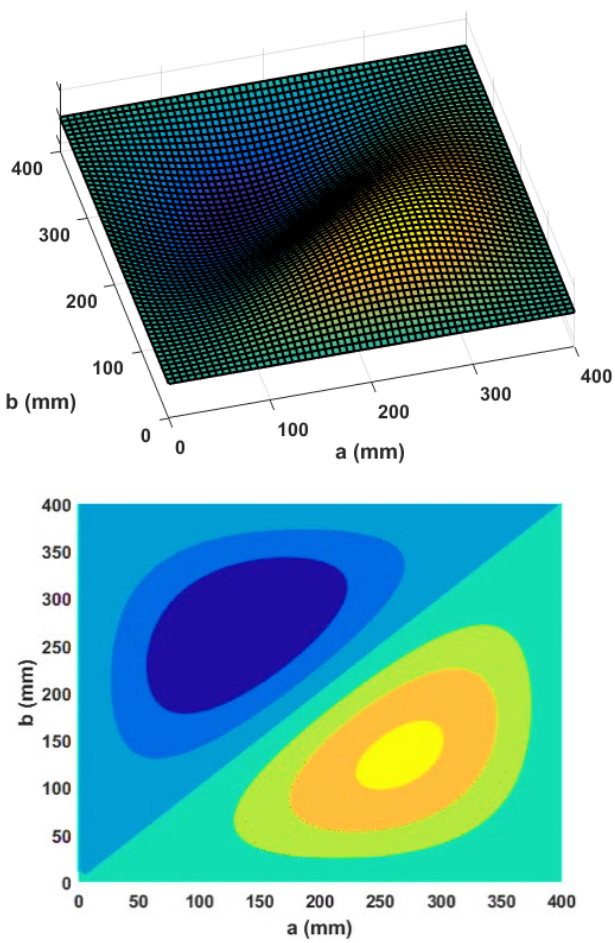


Figure 4: Shear buckling in steel foamed panel without Pasternak foundation effect

foundation effect have been shown in Table 2 and Figure 4. The skew of the buckling mode in the two cases is the same approximately.

In Figure 5, the shear buckling in the steel foamed sandwich panel with foundation effect has been modeled using the Abaqus software. This is for the validation mathematical model used in the current study. The plate is modeled by three isotropic layers. The properties of the plate are in accordance with the above assumptions. Elasticity modulus and shear modulus of solid and foam steel are introduced by using Eq. (4). The results show that the critical shear buckling load and buckling half-waves are similar to the presented equations.

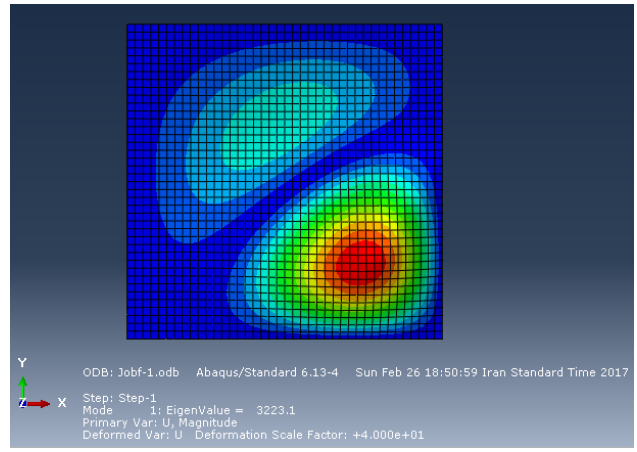


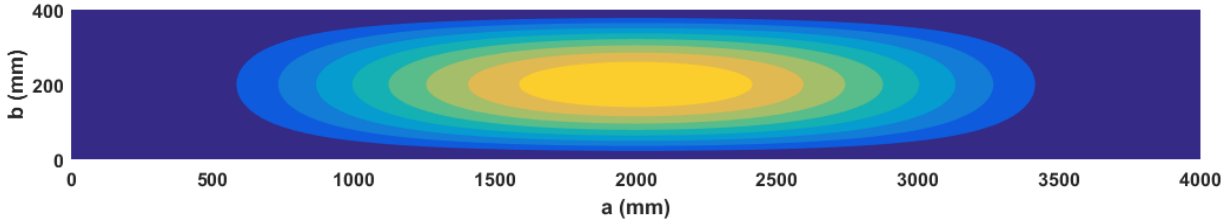
Figure 5: Shear buckling in steel foamed panel without Pasternak foundation effect, modeled in Abaqus

Table 3: Shear buckling in steel foamed panel with $a/b = 10$, with and without Pasternak foundation effect

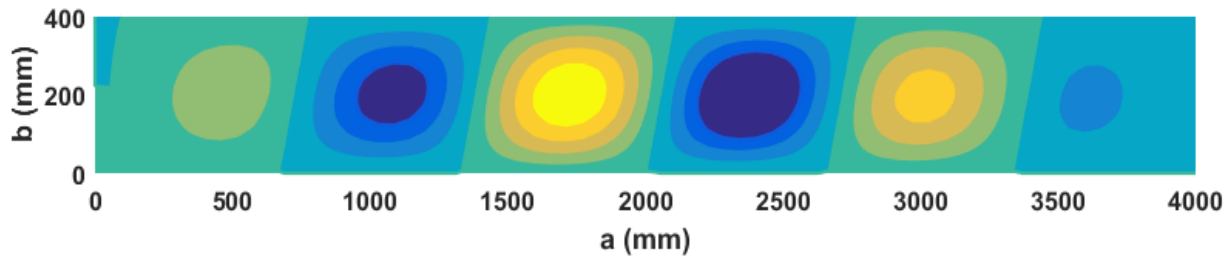
	N_{xy}^0 (N/mm)	φ (degree)	Buckling half-waves
Steel foamed without Pasternak effect	1213	90	1
Steel foamed with Pasternak effect	7555	90	6

Now, it is considered a steel foamed panel with dimension a = 4000 mm and b = 400 mm; thus, the aspect ratio is $a/b = 10$. In Table 3 and Figure 6, the results of shear buckling in steel foamed panel with $a/b = 10$ with and without Pasternak foundation effect are shown. The Pasternak foundation effect causes an increase in critical shear buckling (about 6.2 times with respect to without Pasternak foundation effect) and the buckling half-waves. Also, the skew of the buckling mode in the two cases is the same and equal to 90°.

In Figure 7, critical shear buckling versus aspect ratio of the steel foam sandwich panel with and without Pasternak foundation effect have been shown. In the figure, Case 1 and Case 2 are the panels with α = 0.65, and without and with Pasternak foundation effect, respectively. Case 3 and Case 4 are the panels with α = 0.15, and without and with Pasternak foundation effect, respectively. It can be concluded that in the case without Pasternak foundation effect, by increasing the aspect ratio, the critical shear buckling will decrease. However, the reduction rate is low from 3 to 6. But, in the case with Pasternak foundation effect



(a) Shear buckling in steel foamed panel with $a/b = 10$, without Pasternak foundation effect



(b) Shear buckling in steel foamed panel with $a/b = 10$, with Pasternak foundation effect

Figure 6

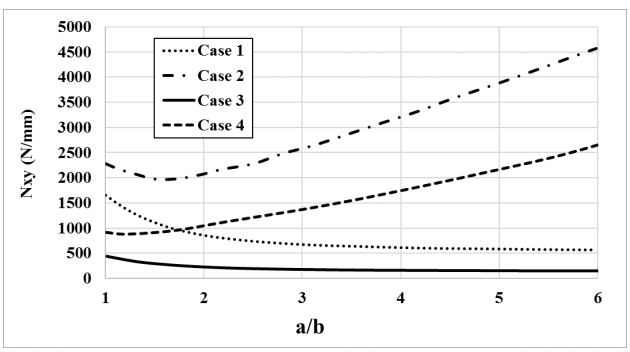


Figure 7: Critical shear buckling versus aspect ratio of the sandwich panel with and without Pasternak foundation effect

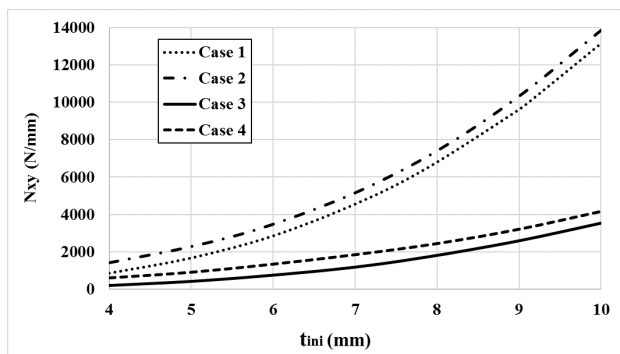


Figure 8: Critical shear buckling versus t_{ini} of the sandwich panel

by increasing the aspect ratio, the critical shear buckling increases severely. The increase is 190% in aspect ratio 6 with respect to aspect ratio 1. Thus, the Pasternak foundation effect is more in high aspect ratio. The results show the Pasternak foundation effect on increasing shear buckling in the panel with $\alpha = 0.65$ is higher than that in the plate with $\alpha = 0.15$.

In Figure 8, critical shear buckling versus t_{ini} of the panel with and without Pasternak foundation effect, and different α have been brought. In the figure, Case 1 and Case 2 are the panels with $\alpha = 0.65$, and without and with Pasternak foundation effect, respectively. Case 3 and Case 4 are the panels with $\alpha = 0.15$, and without and with Pasternak foundation effect, respectively. It is necessary to mention that the aspect ratio is 1. It can be concluded that by increasing t_{ini}, the critical shear buckling increases. However, the increase rate in the cases with $\alpha = 0.65$ is high. The increase rate in the cases without and with Pasternak foundation effect is similar.

4 Conclusion

In this study, critical in-plane shear buckling of steel foam sandwich panel resting on elastic Pasternak foundation was assessed. The panel is a combination of solid steel face sheets and foamed steel cores. The Classic panel theory is employed where their governing equations were solved using the Rayleigh–Ritz method. Uniformly distributed in-plane shear loads were applied to the two opposite edges of the panel and all the four edges of the panel were simply supported.

The results showed that optimum central fraction of the steel foam core would be 65% so that the maximum crit-

ical shear buckling load has taken place. Existing Pasternak foundation cause an increase to the critical shear buckling, so that by increasing the foundation stiffness, the buckling load increases. Often in the aspect ratio equal to 1, for all α, the buckling half-wave is 1 and the skew of buckling mode is 57°. Also, in the case without Pasternak foundation effect, by increasing the aspect ratio, the critical shear buckling will decrease. But, in the case with Pasternak foundation effect, by increasing the aspect ratio, the critical shear buckling increases severely. Thus, the Pasternak foundation effect is more in high aspect ratio. In addition, by increasing t_{ini}, especially in the cases $\alpha = 0.65$ and without and with Pasternak foundation effect, the critical shear buckling increases.

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